

Chirality Changing Phase Transitions in 4d String Vacua

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We provide evidence that some four-dimensional $N = 1$ string vacua with different numbers of generations are connected through phase transitions. The transitions involve going through a point in moduli space where there is a nontrivial fixed point governing the low energy field theory. In an M-theory description, the examples involve wrapped 5-branes leaving one of the ends of the world.

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1. Introduction

There has been great progress in the past two years in unifying string vacua with eight or more supercharges (i.e., the equivalent of 4d $N=2$ supersymmetry). For example, extremal transitions between Higgs and Coulomb phases are believed to connect all type II compactifications on Calabi-Yau threefolds [1,2], while exotic transitions involving “tensionless strings” are believed to connect vacua with different numbers of tensor multiplets in 6d theories with minimal supersymmetry [3].

In studying four dimensional vacua with $N=1$ supersymmetry, one encounters chiral gauge theories. These are very much of interest since the chiral spectrum of quarks and leptons is one of the basic features of the Standard Model. In the context of conventional Lagrangian field theory chirality is preserved by the dynamics, since chiral fermions cannot pick up masses as long as the gauge symmetry remains unbroken. It has been widely believed that string vacua with different net numbers of generations cannot be connected, due to the absence of a conventional field-theoretic mechanism which can provide a mass for generations without similarly giving a mass to anti-generations.

In this paper, we propose that in fact vacua of the $E_8 \times E_8$ heterotic string with 4d $N=1$ supersymmetry and different generation numbers can be connected by phase transitions. These transitions involve going through a point in moduli space where there is no *free* long-distance description of the spacetime physics, i.e. there is a nontrivial fixed point.¹ There is already a familiar analogue of this story in six dimensions. Passing through nontrivial IR fixed points there allows one to change the number of tensor multiplets, despite the absence of a conventional field-theoretic mechanism which can give these multiplets a mass [3]. We argue here that in four dimensions, transitions through similar points seem to allow one to change the net generation number! Other interesting transitions between $N=1$ vacua, which do not involve a change in the net number of generations, have been discussed in [4,5,6,7].

In §2, we describe a class of situations in which we believe such a phase transition occurs. In §3, we study in detail a specific example. We discuss directions for further exploration in §4.

¹ The fact that nontrivial fixed points might play an important role in the explanation of “conifold” singularities in heterotic string theory was suggested some time ago by T. Banks.

2. Wrapped Fivebranes and Chirality Change

2.1. Fibration Picture of the Phase Transition

Consider the $E_8 \times E_8$ heterotic string compactified on a Calabi-Yau threefold X which is a K3 fibration with a section. Let the base \mathbf{P}^1 have radius R . Above the scale $1/R$ the model reduces to the six-dimensional theory obtained by compactifying the heterotic string on K3. We must also specify an appropriate vector bundle \tilde{V} living on the compactification manifold; on K3 this amounts to a configuration of Yang-Mills instantons.

This six-dimensional theory develops a nontrivial fixed point as an instanton shrinks to zero size [3]. As explained in [3], this is a multicritical point. This can be seen by considering the eleven-dimensional structure of heterotic compactifications [8]. The vacuum is a compactification on $S^1/\mathbf{Z}_2 \times K3$, with E_8 gauge fields living on the ends of the interval. The zero-size instanton constitutes a fivebrane on one of the ends. There are two phases connected by the fixed point theory: the Higgs phase is obtained by enlarging the instanton, while a distinct Coulomb-like phase is obtained by moving the fivebrane off into the eleven-dimensional bulk.

The fivebrane worldvolume theory contains a tensor multiplet consisting of a real scalar, an antisymmetric tensor field with self-dual field strength, and fermions. The real scalar's VEV gives the distance of the fivebrane from the end of the world. The perturbative heterotic theory has one tensor multiplet, whose two-form potential combines with that in the 6d gravity multiplet (which has anti-self-dual field strength) to form an unconstrained antisymmetric tensor field. The transition back to the perturbative heterotic theory from the phase with extra tensor multiplets cannot be described in conventional Lagrangian field theory, since the extra tensors cannot pick up masses without having any anti-self-dual partners to couple to.

Of more direct interest in our problem is the fact that the charged matter content also changes in the transition from the perturbative heterotic theory to the theory with the fivebrane in bulk. For definiteness let us take the gauge field configuration of the perturbative heterotic compactification on K3 to consist of an $SU(2)$ bundle embedded in one of the two E_8 s, so that the unbroken gauge symmetry is $E_8 \times E_7$. There are then 45 hypermultiplet singlet bundle moduli and 20 $\frac{1}{2}\mathbf{56}$ s of E_7 . A single small instanton occurs at codimension one in the moduli space. After the transition to the theory with the fivebrane in bulk, there is one fewer $\frac{1}{2}\mathbf{56}$ in the spectrum. This is in accord with the

gravitational anomaly, which requires a loss of 29 hypermultiplets for each tensor multiplet gained.

Let us now consider the theory in four dimensions, i.e. below the scale $1/R$. Again for definiteness let us consider an $SU(3)$ bundle V embedded in one E_8 on the Calabi-Yau threefold X . This leads to E_6 gauge symmetry in spacetime. We will discuss the spectrum in detail (for a set of examples) in the next section. The upshot is that each $\frac{1}{2}\mathbf{56}$ hypermultiplet descends to a set of $\mathbf{27}s$ or to a $\overline{\mathbf{27}}$ in the 4d theory. There is also a set of $\mathbf{27}s$ and one $\overline{\mathbf{27}}$ which do not descend from matter in the 6d theory, i.e. matter which is not associated with the generic fiber $(K3, \tilde{V})$. These are therefore associated with the singular fibers.

In the four-dimensional theory, a zero-size instanton in the generic fiber corresponds to a fivebrane wrapped around the base \mathbf{P}^1 . As shown in [9], it remains consistent with supersymmetry for the fivebrane to move away from the end of the world, since it is wrapped on a holomorphic cycle. In this phase, since one of the $\frac{1}{2}\mathbf{56}s$ has been removed in six dimensions, the corresponding $\mathbf{27}s$ or $\overline{\mathbf{27}}$ have been removed in the four-dimensional theory. We will argue in §3 that no additional states from singular fibers are produced in the transition. So we find that the net generation number has shifted.

In §3.1, we describe how the matter descends from six to four dimensions on the K3 fibration. We will see that the $\frac{1}{2}\mathbf{56}s$ that remain after the 6d phase transition (and hence their descendant $\mathbf{27}s$ or $\overline{\mathbf{27}}s$) can be followed through the transition. The matter that gets removed can also be understood physically. As we will review in §3.2, the small instanton singularity (like all singularities of the worldsheet CFT) can be understood explicitly in a linear sigma model formulation of the worldsheet theory [10]. There, the singularity appears as an extra infinite flat direction in the scalar potential energy. Most of the vertex operators for spacetime states are not supported down this throat, and therefore do not “see” the singularity. The ones that are removed in the transition are supported down the throat.

The question of matter from the singular fibers is considered in §3.3. We study a small instanton singularity (which is present in the generic fiber $(K3, \tilde{V})$ as well as on the singular fibers) in an explicit model. In this example, we show that the small instanton does not intersect the degenerations of \tilde{V} in the singular fibers. This, along with the fact that removing the fivebrane does not lead to additional singularities, leads us to believe that the transition does not produce additional matter states from singular fibers.

One might worry that our proposed transition violates anomaly matching considerations.² In particular, if we ignore nonrenormalizable couplings, there appears to be an anomalous $U(1)_R$ symmetry under which each $\mathbf{27}$ and $\overline{\mathbf{27}}$ transforms with charge $2/3$. There is an anomaly (coming from a triangle diagram with a $U(1)_R$ current and two E_6 currents) which counts the net generation number. It appears that the anomaly will not match on the other side of the transition, since the net generation number has changed. There is, however, another chiral multiplet Φ created in the transition—the dimensional reduction of the 6d tensor multiplet. So we can avoid the problem of matching the anomalies by letting Φ transform under the $U(1)_R$. Then this symmetry is spontaneously broken on the other side of the transition, so the issue does not arise. Another possibility is that the $\mathbf{27}s$ or $\overline{\mathbf{27}}s$ of interest develop anomalous dimensions and naively irrelevant nonrenormalizable terms (which explicitly break the $U(1)_R$ in string theory) are actually important for the physics of the transition.

2.2. Nonperturbative Effects at Lower Scales

So far, we have been discussing the four-dimensional theory below the scale $1/R$ set by the Kaluza-Klein modes on the sphere. There are in fact lower dynamical scales in the problem, set by the strength of worldsheet instanton effects. In the $E_8 \times E_8$ theory, there are two noncritical strings of interest in addition to the usual fundamental heterotic string. These arise from membranes stretched between the fivebrane and the two ends of the world. If ϕ parametrizes the S^1/\mathbf{Z}_2 , then the tension of these noncritical strings goes like ϕ and $(\frac{1}{\alpha'} - \phi)$, respectively. So euclidean worldsheets of these strings wrapping C will generate effects that go like $e^{-R^2 \phi}$ and $e^{-R^2(\frac{1}{\alpha'} - \phi)}$.³ Similar worldsheet instanton effects, associated with noncritical strings which arise in F-theory compactifications on Calabi-Yau fourfolds, have been discussed in [11].

The physics of these instanton effects can be classified according to the splitting type of the vector bundle V over the base $\mathbf{P}^1 C$. Actually, in general there is another vector bundle V_2 embedded in the second E_8 . At the point where the fivebrane is “absorbed”

² We thank the Rutgers theory group, in particular T. Banks and N. Seiberg, for enlightening us on these issues.

³ Some of these ideas were developed in discussions with O. Aharony, O. Ganor, and A. Hanany.

into the second E_8 as an instanton, V_2 is also singular over the base curve C . On general grounds, V and V_2 split as direct sums of line bundles at the singularities of interest

$$V|_C = \sum_{i=1}^r \mathcal{O}(k_i), \quad V_2|_C = \sum_{j=1}^s \mathcal{O}(l_j) \quad (2.1)$$

with $\sum_i k_i = \sum_j l_j = 0$ since $c_1(V) = c_1(V_2) = 0$. We will now discuss various possibilities for physics of the tensionful string worldsheet instantons, sometimes by making use of the T-dual $SO(32)$ string language [12,13]. We have not been exhaustive in our analysis of the possibilities, and expect interesting new physics to arise for some combinations of splitting types of V and V_2 that we have not discussed below.

If $h^0(V|_C \otimes \mathcal{O}(-1))$ and $h^0(V_2|_C \otimes \mathcal{O}(-1))$ are *both* greater than two, then there is no worldsheet instanton generated potential on the ϕ branch. This follows from a standard counting of worldsheet fermion zero modes [14], which are associated with these cohomology groups for the two kinds of noncritical strings. For these splitting types, there are too many zero modes for a potential to be generated. Furthermore, in these cases *fundamental* string worldsheet instantons also cannot remove the “small instanton” point in the classical moduli space, where the nontrivial fixed point occurs. The physics as analyzed at the scale $1/R$ persists in the full quantum theory. Such vacua provide examples where the chirality change can occur while remaining in the moduli space of vacua, i.e. at zero cost in energy.

If $h^0(V|_C \otimes \mathcal{O}(-1))$ or $h^0(V_2|_C \otimes \mathcal{O}(-1))$ is zero, a nonperturbative worldsheet instanton generated superpotential destabilizes the branch with the fivebrane on the interval. One can see this in the E_8 theory by considering tensionful string worldsheet instantons of the first or second E_8 . The number of zero modes on the worldsheet of such a string, for these splitting types, will be small enough for a nontrivial instanton-generated superpotential. This is consistent with known results about $SU(2)$ gauge theories in three dimensions [15,16], which occur in the T-dual $SO(32)$ description.

In the case where $h^0(V|_C \otimes \mathcal{O}(-1)) = 2$ while $h^0(V_2|_C \otimes \mathcal{O}(-1)) = 0$, another interesting phenomenon is known to occur, in some examples, at our small instanton singularity. One finds a pole in the Yukawa couplings of the **27***s* or $\overline{\mathbf{27}}$ *s* which are supported down the throat [17,18]. Before presenting our more detailed evidence for chirality change in §3, we will take a detour in §2.3 to propose a dynamical explanation of the poles in Yukawa couplings in the $E_8 \times E_8$ theory.

We should note that in the latter two cases of splitting types we have discussed, unlike the first case, the chirality change which seems to be possible at scales between the scale μ generated by the instanton superpotential and $1/R$ is obstructed by quantum effects below μ . However, the potential barrier which prevents chirality change in such cases is hierarchically smaller than the string scale. This will become clear in our example in §2.3. This indicates that even in these cases chirality change can “almost” occur, i.e. it is possible at a small cost in energy.

2.3. The Pole in the Yukawa Coupling and Noncritical String Worldsheet Instantons

In [12], the behavior of the $SO(32)$ heterotic string theory on X at a particular small instanton singularity was studied. The non-perturbative resolution of the singularity in that case is given by an $SU(2)$ Yang-Mills theory. As noted in [13], the dynamics of the four-dimensional fixed point theory obtained by compactifying the $E_8 \times E_8$ heterotic string on X at the same type of small instanton singularity is T-dual, after compactification on another circle, to the three-dimensional gauge theory obtained by compactifying the $SO(32)$ heterotic string on $X \times S^1$. In particular, the two theories are related by $r \rightarrow \alpha'/r$ (where r is the radius of the circle) on the locus where Wilson lines have broken the gauge symmetry to $SO(16) \times SO(16)$.

Recently, many aspects of three-dimensional gauge theories have been explained in [16]. The example of [12] involves the $SU(2)$ gauge theory with $N_F = 2$, which has 4 doublets d_i , $i = 1, \dots, 4$. The superpotential is given by

$$W = W_{tree} - Y P f V + e^{-\frac{R^2}{\alpha'}} Y. \quad (2.2)$$

Here for large Y , $Y = e^\Phi$, where Φ is the $SU(2)$ Wilson line. $V_{ij} = \epsilon_{ab} d_i^a d_j^b$ are gauge-invariant combinations of the doublets. A word of explanation is in order concerning the last term in (2.2). In the 3d gauge theory, the last term is $e^{-1/g_4^2} Y = e^{-1/r g_3^2} Y$, where g_3 and g_4 are the three and four-dimensional gauge couplings. The nonperturbative $SU(2)$ gauge coupling in the 4d $SO(32)$ theory is given by $1/g_4^2 = R^2/\alpha'$ [12]. In applying T-duality to that case to study our problem, we must hold R^2/α' constant. Since we also want to study the *four-dimensional* $E_8 \times E_8$ theory, we are also interested in the limit $r \rightarrow 0$, and hence $g_3 \rightarrow \infty$.

In our case, as in [12], the pole will be recovered by taking the tree-level superpotential

$$W_{tree} = V_{12} \mathbf{10}_1 \mathbf{10}_1 \mathbf{1}_{-2} + V_{13}^2 + V_{14}^2 + V_{23}^2 + V_{24}^2 \quad (2.3)$$

Here we are working on the $SO(16)^2$ locus. The gauge symmetry is broken to $SO(16) \times SO(10) \times U(1)$ by the $SU(3)$ vector bundle, and each generation consists of a $(\mathbf{1}, \mathbf{10})_1 + (\mathbf{1}, \mathbf{1})_{-2}$. The first term in (2.3) is proportional to the Yukawa coupling of such a generation. Then integrating out Y , V_{13}, V_{14}, V_{23} , and V_{24} yields $V_{12} = \frac{e^{-\frac{R^2}{\alpha'}}}{V_{34}}$. Plugging this into (2.2) gives

$$W_{dyn} = \frac{\mathbf{10}_1 \mathbf{10}_1 \mathbf{1}_{-2} e^{-\frac{R^2}{\alpha'}}}{V_{34}} \quad (2.4)$$

As we discussed in §2.2, in the original $E_8 \times E_8$ setup, Φ (or more properly $\phi \equiv \Phi/R^2$) gives the tension of the noncritical string which lives on the fivebrane and arises from a membrane stretched from the fivebrane to the end of the world which has the nontrivial $SU(3)$ vector bundle. Similarly, the tension of the string arising from a membrane stretched to the other end is $\frac{1}{\alpha'} - \phi$. The term $e^{-\frac{R^2}{\alpha'}} e^\Phi$ can be understood as coming from worldsheet instantons of this string wrapped around the base \mathbf{P}^1 . This is the analogue for our problem of the fact, emphasized in [16], that the dynamical scale of the 4d gauge theory appears from “twisted instantons” intrinsically in the 3d theory on a circle.

Notice that in the bosonic potential obtained from (2.2), one could take $V = 0$ at a cost in energy of $M_{string} e^{-\frac{R^2}{\alpha'}}$. Then Y can be turned on, leading to a transition at energy scales above $M_{string} e^{-\frac{R^2}{\alpha'}}$ but well below $1/R$. In order to be able to freely move the fivebranes, we would actually need more than one fivebrane in order to cancel the term $|e^{-\frac{R^2}{\alpha'}} \sum_i Y_i|^2$ in the potential energy. Here, Y_i determines the position of the i th fivebrane. It is not difficult to find examples where several fivebranes can move off into the interval giving such a potential – the example we discuss in §3 is of this type.

3. An Explicit Example

In this section, we will explain in some detail the relation described in §2.1 between the six and four-dimensional spectra by using standard techniques for the analysis of string compactifications. We do this in the context of a concrete example. The example we discuss below is of the class which involves smooth chirality change at the scale $1/R$ but in which there is a potential generated by instantons. However, we expect the analysis to generalize to a large class of K3 fibrations and gauge bundles.

3.1. Fibration of Charged Matter

Consider the (0,2) model which is obtained as a deformation of the (2,2) model on the hypersurface of degree 8 in \mathbf{WP}_{11222}^4 . This is a K3 fibration, with the K3 fiber realized as a hypersurface of degree 4 in \mathbf{P}^3 . This model was studied in detail in [12] using a gauged linear sigma model on the worldsheet [10], and we will recall from that analysis what we need here. Let us denote the projective coordinates on \mathbf{WP}_{11222}^4 by z_1, \dots, z_5 . The hypersurface is given by the vanishing locus of a degree 8 polynomial $G(z_1, \dots, z_5)$. For definiteness, let us take

$$G(z_1, \dots, z_5) = \frac{z_1^8}{8} + \frac{z_2^8}{8} + \frac{z_3^4}{4} + \frac{z_4^4}{4} + \frac{z_5^4}{4}. \quad (3.1)$$

Setting $z_1 = \lambda z_2$ and $y = z_2^2$, we find the defining equation for the K3 fiber:

$$\tilde{G}(y, z_3, z_4, z_5) = (1 + \lambda^8) \frac{y^4}{8} + \frac{z_3^4}{4} + \frac{z_4^4}{4} + \frac{z_5^4}{4}. \quad (3.2)$$

The bundle is specified by giving polynomials J_i , $i = 1, \dots, 5$ of degrees (7,7,6,6,6) respectively. On the (2,2) locus, the vector bundle becomes the tangent bundle, so that $J_i = \frac{\partial G}{\partial z_i}$. The (0,2) deformation can be obtained by deforming these polynomials:

$$\begin{aligned} J_1 &= p[z_1^7 + G_1] \\ J_2 &= p[z_2^7 + G_2] \\ J_3 &= p[z_3^3 + G_3] \\ J_4 &= p[z_4^3 + G_4] \\ J_5 &= p[z_5^3 + G_5] \end{aligned} \quad (3.3)$$

Then the vector bundle on the K3 fiber is given by the polynomials

$$\begin{aligned} F_1 &= \frac{\lambda J_1 + J_2}{z_2} \Big|_{z_1=\lambda z_2, y=z_2^2} = p[(1 + \lambda^8)y^3 + \tilde{G}_1] \\ F_2 &= J_3(y, z_3, z_4, z_5) = p[z_3^3 + \tilde{G}_3] \\ F_3 &= J_4(y, z_3, z_4, z_5) = p[z_4^3 + \tilde{G}_4] \\ F_4 &= J_5(y, z_3, z_4, z_5) = p[z_5^3 + \tilde{G}_5] \end{aligned} \quad (3.4)$$

The charged matter ($\frac{1}{2}\mathbf{56}_s$) in six dimensions arises partly from monomials of degree 4 in y, z_3, z_4, z_5 modulo the F_i . There are 6 such monomials A_2 of the form $y^2 P_2(z_3, z_4, z_5)$,

7 monomials A_1 of the form $yP_3(z_3, z_4, z_5)$, and 6 monomials A_0 of the form $P_4(z_3, z_4, z_5)$, where we have indicated by $P_d(z_i)$ a polynomial of degree d in z_i . This gives $19 \frac{1}{2} \mathbf{56}s$. The index theorem ensures that there are 20; the additional one will be explained below. As explained in [19], we can read off the transformation properties of these states under monodromy around points on the base \mathbf{P}^1 above which the fiber theory is singular. Our situation is a generalization of that studied in [19] since in the (0,2) context physical singularities occur when the *vector bundle* degenerates, regardless of the behavior of the manifold at such points. It will simplify our analysis to consider a choice of polynomials such that none of F_2, F_3 or F_4 contain a term proportional to y^3 . Then singularities occur when $1 + \lambda^8 = 0$.

Around the points on the base \mathbf{P}^1 with $1 + \lambda^8 = 0$, y has monodromy $y \rightarrow e^{\frac{2\pi i}{4}} y$. This leads to monodromies for the $\frac{1}{2} \mathbf{56}s$ (coming from monomials A_2, A_1 , and A_0) that we found above. These transform under the monodromy as $A_q \rightarrow e^{\frac{2\pi i q}{4}} A_q$. This means that near a monodromy point λ_0 , $A_q \sim (\lambda - \lambda_0)^{\frac{q}{4}}$. Hence, taking into account all eight singular fibers, we see that A_q behaves like a polynomial of degree $2q$ in λ . Therefore we expect $2q + 1$ $\mathbf{27}s$ in the 4d theory for each $\frac{1}{2} \mathbf{56}$ corresponding to a monomial A_q in the 6d theory.

This is indeed what one finds from the analysis of charged matter in the theory compactified on the full threefold X . In this case we must count monomials of degree 8 in z_1, \dots, z_5 modulo the J_i . We find $6 \cdot 5 = 30$ monomials of the form $P_4(z_1, z_2)P_2(z_3, z_4, z_5)$, $7 \cdot 3 = 21$ of the form $P_2(z_1, z_2)P_3(z_3, z_4, z_5)$, and 6 of the form $P_4(z_3, z_4, z_5)$. These correspond to the expected descendants of A_2, A_1 , and A_0 respectively. There are additional states in the 4d theory which do not arise from the $\mathbf{56}s$ present in the six-dimensional theory obtained by compactification on the generic fiber. Some of these correspond to the 21 monomials of the form $P_6(z_1, z_2)P_1(z_3, z_4, z_5)$ and the 5 monomials of the form $P_8(z_1, z_2)$. There are also 3 non-deformation-theoretic $\mathbf{27}s$.

What about the $\overline{\mathbf{27}}s$? One way to understand these is to use the linear sigma model description reviewed in [12]. There the coordinates z_1, \dots, z_5 are scalar fields transforming with charges $(1, 1, 1, 1, 1)$ and $(1, 1, 0, 0, 0)$ under two $U(1)$ gauge symmetries on the worldsheet. There are, in addition, scalar fields σ_1 and σ_2 which become part of the (2,2) gauge multiplets on the (2,2) locus. On the (2,2) locus, the gauge multiplets correspond to Kahler moduli. Each gauge multiplet yields one $\overline{\mathbf{27}}$ (whose vertex operator is proportional to σ [18]). So in our model, there are two $\overline{\mathbf{27}}s$, one coming from the generic fiber theory

and the other coming from the base \mathbf{P}^1 . The contribution in the generic fiber gives the requisite $20th \frac{1}{2}\mathbf{56}$ in the 6d theory.

So we see that each $\frac{1}{2}\mathbf{56}$ present in the 6d theory leads to *either* a set of $\mathbf{27}s$ or a $\overline{\mathbf{27}}$ in the 4d theory. This suggests, as discussed in §2, that in a transition in which a single $\frac{1}{2}\mathbf{56}$ is removed, the corresponding charged matter in 4d is removed. This charged matter being chiral, the phase transition then changes the net generation number of the 4d theory (modulo contributions from the singular fibers to be discussed below).

This statement depends on being able to follow the remaining $\frac{1}{2}\mathbf{56}s$ through the transition in the 6d theory. We will now discuss the localization of matter both in F-theory and in the heterotic linear sigma model description.

3.2. Localization of 6d Matter

The 6d theory is amenable to analysis using F-theory [20].⁴ There the heterotic compactification on K3 with instanton numbers $12+n$ and $12-n$ in the two E_8 s corresponds to compactification of F-theory on an elliptic Calabi-Yau threefold K with base F_n [21]. The manifold K consists of a T^2 varying over F_n (which is itself a \mathbf{P}^1 bundle over \mathbf{P}^1). Its defining equation is

$$y^2 = x^3 + f(w_1, w_2)x + g(w_1, w_2). \quad (3.5)$$

This describes a T^2 at each point (w_1, w_2) on the base F_n . In general, one has

$$\begin{aligned} f(w_1, w_2) &= \sum_{k=-4}^4 f_{8-nk}(w_2)w_1^{4-k} \\ g(w_1, w_2) &= \sum_{k=-6}^6 g_{12-nk}(w_2)w_1^{6-k} \end{aligned} \quad (3.6)$$

where the subscripts on the polynomials f_{8-nk} and g_{12-nk} indicate their degrees. The discriminant $\Delta = 4f^3 + 27g^2$ vanishes at singularities of the T^2 fiber.

In our case, we have an $SU(2)$ bundle in one of the E_8 s, which leaves an unbroken $E_7 \times E_8$ gauge symmetry in spacetime. On the F-theory side, we must therefore induce an E_7 singularity on the “top” \mathbf{P}^1 of the F_n . Let us put this singularity at $w_1 = 0$. As

⁴ F-theory can also be applied to the study of 4d $N = 1$ theories, but it remains to understand charged matter in such compactifications.

explained in [21], this requires $f \sim w_1^3$ and $g \sim w_1^5$ near the singularity. So our polynomials (3.6) get restricted to

$$\begin{aligned} f(w_1, w_2) &= \sum_{k=-4}^1 f_{8-nk}(w_2) w_1^{4-k} \\ g(w_1, w_2) &= \sum_{k=-6}^1 g_{12-nk}(w_2) w_1^{6-k} \end{aligned} \quad (3.7)$$

Near the singularity, the discriminant looks like

$$\Delta \sim w_1^9 (f_{8-n}(w_2) + \mathcal{O}(w_1)) \quad (3.8)$$

As explained in [22,23], the $\frac{1}{2}\mathbf{56}s$ of E_7 are associated with zeroes of the polynomial f_{8-n} . In the case with all instantons in one E_8 which we have been studying, $n = -12$ and there are 20 $\frac{1}{2}\mathbf{56}s$. The E_8 symmetry of the other “end of the world” is located at $w_1 = \infty$.

We would now like to study the phase transition, which involves inducing a small instanton and moving the resulting fivebrane off the end of the interval. As explained in [21,24], this is realized in F theory by the blowing up of a point on the F_n . Following §6.1 of [21], we must add a new coordinate u , and mod out by an additional rescaling under which

$$(x, y, w_1, w_2, u) \rightarrow (\lambda^2 x, \lambda^3 y, \lambda w_1, \lambda w_2, \lambda^{-1} u). \quad (3.9)$$

This restricts our polynomials to be of the form

$$\begin{aligned} f(w_1, w_2, u) &= w_1^3 \sum_{l=1}^{8-n} f_{1l} w_2^l u^{l-1} + \mathcal{O}(w_1^4) \\ g(w_1, w_2) &= w_1^5 \sum_{l=1}^{12-n} g_{1l} w_2^l u^{l-1} + \mathcal{O}(w_1^6) \end{aligned} \quad (3.10)$$

In particular, in order to induce the small instanton (which makes possible the blow-up), we had to restrict the polynomials f and g so that $f_{8-n}(w_2) \sim w_2 \tilde{f}_{7-n}$ and $g_{12-n}(w_2) \sim w_2 \tilde{g}_{11-n}$. The blow-up removes the point $w_1 = w_2 = 0$, and one is left with $7 - n$ $\frac{1}{2}\mathbf{56}s$ at the zeroes of \tilde{f}_{7-n} . Thus the $\frac{1}{2}\mathbf{56}s$ which remain can be followed continuously through the transition.

The heterotic linear sigma model description gives a simple physical way to characterize the matter fields which get removed during the transition (at least those coming from

the generic fiber). The bosonic potential looks like

$$\begin{aligned}
& \frac{e_1^2}{2} \left(|z_1|^2 + |z_2|^2 + 2|z_3|^2 + 2|z_4|^2 + 2|z_5|^2 - 8|p|^2 - r_1 \right)^2 \\
& + \frac{e_2^2}{2} \left(|z_1|^2 + |z_2|^2 - 2|x|^2 - r_2 \right)^2 \\
& + |\sigma_1 + \sigma_2|^2 \left(|z_1|^2 + |z_2|^2 \right) + |\sigma_1|^2 \left(64|p|^2 + 4|z_3|^2 + 4|z_4|^2 + 4|z_5|^2 \right) \\
& + 4|\sigma_2|^2|x|^2 + |G|^2 + \sum_i |J_i|^2 + |J_x|^2
\end{aligned} \tag{3.11}$$

Here p and x are scalar fields which decouple in the generic large-radius phase, J_x is a polynomial constrained to satisfy $z_1 J^1 + z_2 J^2 = 2x J_x$, and $e_{1,2}$ are the gauge couplings of the two worldsheet $U(1)$ gauge groups. As explained in [10], the singularities of the IR conformal field theory have a beautiful realization in this linear model.

One type of singularity occurs for special values of the moduli $r_{1,2}$. In particular, when $r_1 = r_2$, there arises a new flat direction in (3.11) in which σ_1 and σ_2 can go off to infinity as long as $\sigma_1 + \sigma_2 = 0$. On this branch, z_3, z_4, z_5, p , and x are frozen at zero. Recall that the vertex operator for the $\overline{\mathbf{27}}$ which comes from a $\frac{1}{2}\mathbf{56}$ in six dimensions is given by $\sigma_1 - \sigma_2$. So the wavefunction for one of the 4d $\overline{\mathbf{27}}s$ is supported along the “throat” which produces the singularity. On the other hand, the $\mathbf{27}s$ coming from $\frac{1}{2}\mathbf{56}s$ in the 6d theory arise from monomials involving z_3, z_4 , and z_5 . These are frozen at zero way out on the throat where σ_1 and σ_2 diverge. These can therefore not be affected by the transition involving the fivebrane.

A similar distinction between the types of states arises at singularities obtained by specializing the parameters in the J_i to induce a singularity on the generic fiber. Then for example z_3 and p might diverge along a new branch in (3.11), forcing $\sigma_1 = 0$. So only (a subset of) the $\frac{1}{2}\mathbf{56}s$ which lead to $\mathbf{27}s$ can be removed in the phase transition.

3.3. The Singular Fibers

We would now like to give the promised explicit verification, in an example, that the small instanton does not intersect the singularity associated with the singular fibers. In our model (3.4), we have singularities at $\lambda = \lambda_0$ such that $1 + \lambda_0^8 = 0$, since there the F_i can all vanish without y vanishing. These λ_0 are the points on the base \mathbf{P}^1 over which there are singular fibers. The singularity in the fiber theory occurs at $z_3 = z_4 = z_5 = 0$. Let us take for simplicity $\tilde{G}_1 = yz_3^2$, $\tilde{G}_3 = e^{-\frac{i\pi}{2}} z_3 z_4^2 - y^2 z_3$, and $\tilde{G}_4 = -e^{-\frac{i\pi}{2}} z_4 z_3^2$ with \tilde{G}_5

vanishing. Then there are singularities of the bundle (which amount to some collection of small instantons) at $y = z_5 = 0; z_3 = e^{\frac{i\pi k}{4}} z_4$ (for $k = 1, 3, 5, 7$). These small instanton singularities do not intersect the singularity $z_3 = z_4 = z_5 = 0$ of the singular fiber. As explained in §3.2, the removal of a small instanton from a fiber K3 does not seem to introduce any additional singularities. For instance, in the F-theory description which renders the whole analysis geometrical, the removal of the fivebrane corresponds to blowing up a point and does not introduce new singularities. Hence, it appears very implausible that removing the small instanton fivebranes could change the contributions from the singular fibers.

Because of the subtleties involving the singular fibers, it would be very interesting to develop techniques for describing directly the vector bundle which remains on the end of the world after the transition. Perhaps this can be done using appropriate generalizations of [25,5] to chiral models.

4. Discussion

We presented evidence here that in a specific class of M-theory vacua – compactifications on S^1/\mathbf{Z}_2 times a K3 fibration – there seem to be chirality-changing phase transitions. It is of significant interest for the vacuum degeneracy problem to understand the extent to which this phenomenon occurs in more general vacua.

In particular, heterotic compactifications typically develop pointlike singularities at some codimension in moduli space. The nonperturbative physics at such singularities has yet to be determined. It was shown long ago that naively, (2,2) models can be connected at such singularities [1]. The Euler character χ of the manifold changes in the transition. In context of type II string theories, χ gives the difference between the numbers of hypermultiplets and vector multiplets in the resulting 4d $N = 2$ theory. The transitions of [1] correspond, as explained in [2], to transitions between Higgs and Coulomb phases in 4d $N = 2$ gauge theory.

In the context of the heterotic string, χ gives the net generation number for (2,2) models, and the transitions of [1] cannot occur within conventional (weakly coupled) Lagrangian field theory. In the example discussed in this paper, the occurrence of a nontrivial fixed point theory enables branches with different numbers of generations to meet. If the nonperturbative physics which resolves the “conifold” singularities of [1] in heterotic

string compactifications also involves nontrivial interacting fixed points, one can reasonably speculate that branches of the moduli space with different numbers of generations may be meeting there as well.⁵

It is attractive to conjecture that there is a web of heterotic (0,2) and (2,2) vacua, much like the web of $N=2$, $d=4$ theories, connected by transitions through nontrivial fixed points (such as those found in [26]). In fact, given that there are already known transitions which connect (0,2) heterotic vacua on topologically distinct manifolds (but at fixed generation number) purely *within* perturbative string theory [7], it is very interesting to continue the study of nonperturbative transitions between (0,2) models with different generation numbers on a *fixed* Calabi-Yau manifold.

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⁵ Note that unlike the case studied here, where the transition is mediated by emission of a wrapped M theory fivebrane, it is difficult to see how one would understand directly what is on the “other side” of possible transitions in the pointlike conifolds.

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